# CATHODE ELECTRIC-FIELD-INTENSITY-DISTRIBUTION FUNCTION

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The behavior of the distribution function for the electric field intensity at the cathode is considered including only nearest-neighbor effects and is compared with the behavior of the distribution function obtained when including the effects of many ions. Motion of ions in the near-cathode region and their nonuniform density there are taken into account in the calculations of the distribution function. It is shown that for a broad range of parameters the resultant distribution function differs little from the distribution function found when constant density is assumed.

A method was proposed [1] for taking into account the effect on the emission characteristics of hot cathodes produced by fluctuating microfields created in the average background field because of the motion of individual ions near the emitting surface. This can be accomplished by averaging the thermoemission current density $j_0$ calculated from the Richardson–Dushman formula with the Schottky correction over the distribution function $f(E)$ for the electric field E at the cathode surface. This distribution function $f(E)$ was found [2] and averaging of the quantity $j_0$ over it performed:

$$\langle j\rangle = \int j_0(E) f(E)\, dE \qquad (j_0 = AT^2 \exp(-e\varphi_0/kT + e\sqrt{eE}/kT))$$

Here, A is the thermoemission constant, e is the charge on the electron, k is the Boltzmann constant, T is the cathode temperature, and $\varphi_0$ is the work function.

It was assumed [2] first that the field intensity at a given point of the cathode depended on the location of the ion closest to it, and second that the ion density n in the near cathode region of the discharge was constant. We discuss these assumptions in greater detail.

1. We show that the value of the distribution function $f(E)$ for large values of E is determined by the location of the closest ion.

Let the point $(x_0, y_0)$ be on the surface of the cathode. We describe a hemisphere of radius R around it and assume that N ions are incident on this hemisphere. The magnitude of the electric field created by these ions at the point $(x_0, y_0)$ is

$$\mathbf{E} = \sum_{k=1}^{N} \frac{q_k \mathbf{r}_k}{|\mathbf{r}_k|^3} = \sum_{k=1}^{N} \mathbf{E}_k$$

We consider the field at the cathode surface without including mirror reflection of the ions, and, in addition, we consider the field intensity vector **E** and not its normal component $E_Z$. This leads to a change in the coefficients in the distribution function, but its fundamental nature is not changed.

We further assume $N \to \infty$ when $R \to \infty$, where $N/(^2/_3)\pi R^3 = n$ and n is the ion density in the main plasma volume; we then set n = const and show that for variable n the basic relations are also fundamentally unchanged.

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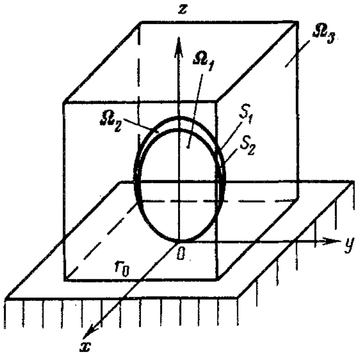


Fig. 1

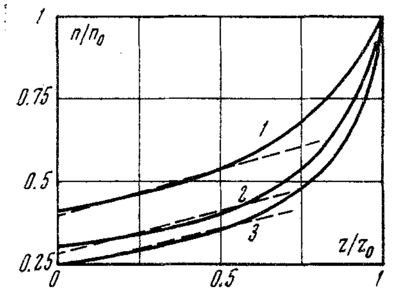


Fig. 2

We find the probability that **E** is in the range ($\mathbf{E}_0$, $\mathbf{E}_0 + d\mathbf{E}$). Using the Markov method described in detail in [3], we have

$$f_N(\mathbf{E}_0) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \exp(i\rho\mathbf{E}_0) A_N(\rho)\, d\rho$$

$$A_N(\rho) = \sum_{k=1}^{N} \int_{|r_k|=0}^{R} \exp(i\rho\mathbf{E}_k)\, \tau_k(\mathbf{r}_k)\, d\mathbf{r}_k$$

where $\tau_k(\mathbf{r}_k)$ is the distribution which determines the probability that the k-th ion has the coordinate $\mathbf{r}_k$. We further assume that only those fluctuations occur which are compatible with a constant mean density, i.e., $\tau_k(\mathbf{r}_k) = 2/3\, \pi R^3 \tau(q)$, where $\tau(q)$ is the frequency at which ions are encountered. Then

$$A_N(\rho) = \left[\frac{3}{2\pi R^3} \int_{|\mathbf{r}|=0}^{R} \exp(i\rho\varphi)\, \tau(q)\, d\mathbf{r}\right]^N \qquad (\varphi = q\mathbf{r} / |\mathbf{r}|^3)$$

Passing to the limit for $R \to \infty$, $N \to \infty$, we have

$$f(\mathbf{E}) = \frac{1}{4\pi^3} \int_{-\infty}^{\infty} \exp(-i\rho\mathbf{E}) A(\rho)\, d\rho$$

$$\left( A(\rho) = \lim_{R\to\infty} \left[\frac{3}{2\pi R^3} \int_{|\mathbf{r}|=0}^{R} \exp(i\rho\varphi)\, \tau(q)\, d\mathbf{r}\right]^{2\pi R^3 n/3} \right)$$

Calculating $A(\rho)$, we have

$$A(\rho) = \exp\left[-\frac{2n}{15}(2\pi q)^{3/2} |\rho|^{3/2}\right]$$

and then

$$f(\mathbf{E}) = \frac{1}{2\pi^2 |\mathbf{E}|^3} \int_0^{\infty} \exp\left(-\frac{a x^{3/2}}{|\mathbf{E}|^{3/2}}\right) x \sin x\, dx$$

$$(x = |\rho|\,|\mathbf{E}|, \quad a = {}^4/_{15}(2\pi q)^{3/2} n/2)$$

The asymptotic behavior of the distribution $f(\mathbf{E})$ when $[\mathbf{E}] \to \infty$ has the form

$$f(\mathbf{E}) = \pi q^{3/2} n\, |\mathbf{E}|^{-5/2} \tag{1.1}$$

The main portion of the thermoemission current is determined by large values of E. The distribution (1.1), which was obtained by consideration of all ions near a given point on the cathode, agrees except for a factor of 1/2 with the asymptotic behavior of the distribution (12) in [2], which was obtained considering only the effect of the nearest ion. Thus the first assumption is valid and the distribution (1.1) can be used for calculation of thermoemission current density.

2. We consider in greater detail the second assumption about the constancy of the density in the near-cathode region. In reality, the ion density varies because of the cathode potential drop with the consequence

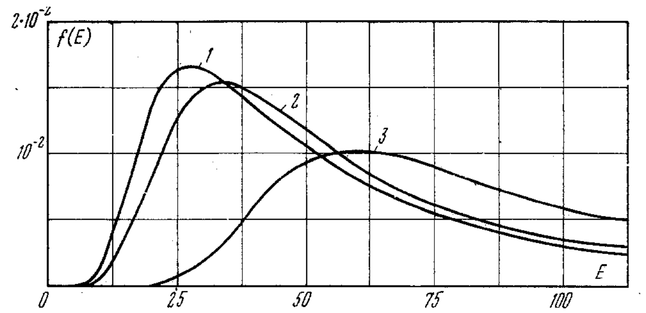


Fig. 3

that the distribution function for the electric field intensity at the surface of the cathode also varies. We determine the form of the distribution function $f(E)$ including variable ion density. We locate the origin of the coordinates at the point on the cathode surface under consideration (Fig. 1). The x and y axes are in the plane of the cathode, and the z axis is perpendicular to it. We consider the surfaces $S_1$ and $S_2$, the equations for which are

$$E = 2qz\,(x^2 + y^2 + z^2)^{-3/2}, \quad E - dE = 2qz\,(x^2 + y^2 + z^2)^{-3/2}$$

Since the distribution function for the electric field intensity at the point (0, 0) is determined by the position of the nearest ion, then in order that the normal component of the electric field fall within the range $(E_0 - dE, E_0)$, it is necessary there be no ion within the region $\Omega_1$ and one ion in the region $\Omega_2$, i.e.,

$$p\,(E_0 - dE \leqslant E_z \leqslant E_0) = f\,(E)\,dE = p_1\,(0 \in \Omega_1)\,p_2\,(1 \in \Omega_2)$$

These considerations are similar to those in [2] for the case with constant density with the exception the probabilities $p_1$ and $p_2$ are written in somewhat different form.

We consider the variation of ion density in the near-cathode region of the discharge. It was shown [4, 5] that over a broad range of discharge parameters, the variation in potential in the near-cathode region is almost linear. Assuming that the ion velocity v is related to the potential difference $\Delta U = U_0 - U$ experienced by the ion through the expression

$$v = (v_0^2 + \Delta U 2e\,/\,m)^{1/2},$$

that the ions in the near-cathode region are nonrelativistic, and that the law for variation of potential is of the form $U = U_0 z/z_0$, where z is the distance from the cathode and $z_0$ is the thickness of the layer in which the near cathode potential drop $U_0$ occurs, we obtain

$$n = n_0\,(^1/_2 m v_0^2)^{1/2}\,[^1/_2 m v_0^2 + eU_0\,(1 - z\,/\,z_0)]^{-1/2} \tag{2.1}$$

The relation (2.1) is shown graphically in Fig. 2 for various values of $U_0$ and for an initial ion velocity $v_0$ corresponding to a plasma temperature of 1 eV. The curves 1, 2, and 3 correspond to $U_0 = 5$, 10, and 15 V.

As is clear from Fig. 2, the ion density relative to the density in the central region of the discharge varies only by a small factor for a plasma temperature of ~ 1 eV and a cathode drop of ~ 10 V with the variation of density being nearly linear close to the cathode surface.

Knowing the law for variation of density, we write down expressions for the probabilities $p_1$ and $p_2$.

The weight $V_1$ assigned to the region $\Omega_1$ can be calculated in the following manner:

$$V_1 = \pi \int_0^{\sqrt{2q/E}} \left[\left(\frac{2qz}{E}\right)^{2/3} - z^2\right] n\,(z)\,dz \tag{2.2}$$

In order to obtain an expression suitable for practical calculations, we write the expression for n(z) in the form

$$n\,(z) = n_0 + \gamma z \tag{2.3}$$

The relation (2.3) is shown graphically in Fig. 2 by the dashed lines. Substituting the expression for n(z) from Eq. (2.3) into Eq. (2.2), we have

$$V_1 = \frac{4\pi n_0}{15}\left(\frac{2q}{E}\right)^{3/2} + \frac{\pi\gamma}{8}\left(\frac{2q}{E}\right)^2$$

Calculating in similar fashion the weights assigned to the regions $\Omega_2$ and $\Omega_3$, we have

$$V_2 = \left[\frac{2\pi n_0}{5}\frac{(2q)^{3/2}}{E^{5/2}} + \frac{\pi\gamma (2q)^2}{4E^3}\right] dE, \qquad V_3 = 8r_0^3(n_0 + \gamma r_0)$$

We obtain for the probabilities $p_1$ and $p_2$

$$p_1 = \exp\left[-\frac{2\pi n_0}{15}\left(\frac{2q}{E}\right)^{3/2} - \frac{\pi\gamma}{8}\left(\frac{2q}{E}\right)^2\right], \quad p_2 = \left[\frac{2\pi n_0}{5}\frac{(2q)^{3/2}}{E^{5/2}} + \frac{\pi\gamma (2q)^2}{4E^3}\right] dE$$

The distribution function for the normal component of the electric field intensity has the form

$$f(E) = \left(\frac{2\pi n_0 (2q)^{3/2}}{5E^{5/2}} + \frac{\pi\gamma (2q)^2}{4E^3}\right) \exp\left[-\frac{4\pi n_0}{15}\left(\frac{2q}{E}\right)^{3/2} - \frac{\pi\gamma (2q)^2}{8E^2}\right] \quad (2.4)$$

The relation $f(E)$ is shown in Fig. 3 for $n = 10^{16}$ cm$^{-3}$ and for values of $\gamma$ equal to 0, $10^{21}$, and $10^{22}$ cm$^{-4}$ for curves 1, 2, and 3.

We calculate the value of the thermoemission current averaged over the distribution (2.4),

$$\langle j \rangle = \int_0^{E_*} j_0(E) f(E)\, dE = I_1 + I_2 =$$
$$= \int_0^{E_*} j_0(E) \frac{2\pi n_0 (2q)^{3/2}}{5E^{5/2}} \exp\left[-\frac{4\pi n_0}{15}\left(\frac{2q}{E}\right)^{3/2} - \frac{\pi\gamma (2q)^2}{8E^2}\right] dE +$$
$$+ \int_0^{E_*} j_0(E) \frac{\pi\gamma (2q)^2}{4E^3} \exp\left[-\frac{4\pi n_0}{15}\left(\frac{2q}{E}\right)^{3/2} - \frac{\pi\gamma (2q)^2}{8E^2}\right] dE \quad (2.5)$$

Determining $I_1$ by neglecting the second term in the exponential for large E, we have the approximate expression

$$I_1 = AT^2 \exp\left(-\frac{e\varphi_0}{kT}\right)\left[1 + \frac{4\pi n_0 kT}{5E_*^2} \exp\left(\frac{e\sqrt{eE_*}}{kT}\right)\right] \quad (2.6)$$

Equation (2.6) agrees with the leading portion of expression (14) in [2]. This expression describes the component of the thermoemission current from the cathode which corresponds to a constant ion density $n_0$ in the cathode layer.

We determine $I_2$, neglecting the second term in the exponential

$$I_2 = AT^2 \exp\left(-\frac{e\varphi_0}{kT}\right) \frac{\pi\gamma q^2}{4} 8 \left(\frac{e\sqrt{e}}{kT}\right)^4 \exp\left(\frac{e\sqrt{eE_*}}{kT}\right)\left(\frac{e\sqrt{eE_*}}{kT}\right)^{-5} \quad (2.7)$$

Consequently,

$$\langle j \rangle = AT^2 \exp\left(-\frac{e\varphi_0}{kT}\right)\left\{1 + \left(\frac{4\pi n_0 kT}{5E_*^2} + \frac{2\pi\gamma\sqrt{e}\,kT}{E_*^{5/2}}\right)\exp\left(\frac{e\sqrt{eE_*}}{kT}\right)\right\} \quad (2.8)$$

We investigate the case where the second term in the braces in Eq. (2.8) is dominant:

$$\frac{4\pi n_0 kT}{5E_*^2} > \frac{2\pi\gamma\sqrt{e}\,kT}{E_*^{5/2}},$$

i.e., for

$$\frac{n_0}{\gamma} > \frac{5}{2}\sqrt{\frac{e}{E_*}} \quad (2.9)$$

the third term in Eq. (2.8) can be neglected in comparison with the second. Since in cases of practical interest $n_0/\gamma \sim 10^{-6}$, the relation (2.9) is satisfied to a high degree of accuracy and Eq. (2.8) agrees with the expression for the thermoemission current density obtained under the assumption the ion density in the near-cathode region is constant.

Thus, the maximum ion density gradient near the cathode has been evaluated. If the density gradient satisfies the condition (2.9) under actual conditions, the variation of n need not be taken into account; otherwise one should use an expression such as Eq. (2.8).

Thus, in the determination of the thermoemission current density and of the asymptotic behavior of the distribution function for the normal component of the electric field at the cathode, there is no need to include the effect of an ensemble of particles since j and $f$ (E) are determined by the distribution function for the nearest neighbor. Over a broad range of discharge parameters, it is sufficient to know the ion density in the immediate neighborhood of the discharge in order to determine the thermoemission current density. The density subsequently changes but has practically no effect on the magnitude of the thermoemission current. Only the ion closest to a given point on the cathode has a decisive effect on the magnitude of E.

It was assumed the condition $n_e \ll n_i$ was satisfied. For $s = j_i/j_e \gtrsim 0.1$, i.e., when the fractional ion current is significant (a situation which is realized in a gas discharge), this condition is well satisfied because the electron velocity close to the cathode surface is $\sim 10^8$ cm/sec while the velocity of ions which acquire energy in the cathode jump is no more than $10^6$ cm/sec.

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